A quasi-two-dimensional depth-dependent mobility model suitable for device simulation for Coulombic scattering due to interface trapped charges

Siddharth Potbhare,a) Neil Goldsman,b) and Gary Pennington
Department of Electrical and Computer Engineering, University of Maryland, College Park, Maryland 20742

Aviars Lelis
U.S. Army Research Laboratory, 2800 Powder Mill Road, Adelphi, Maryland 20783

James M. McGarrity
Berkeley Research Association, Springfield, Virginia 22150

(Received 14 February 2006; accepted 26 May 2006; published online 31 August 2006)

The silicon carbide (SiC)–silicon dioxide (SiO2) interface in SiC metal-oxide-semiconductor field-effect transistors (MOSFETs) has shown the presence of a very large number of trap states. These traps become filled during inversion causing a lowering of conduction charge in the inversion layer, and increases Coulombic scattering of mobile charges. Owing to the large number of occupied interface traps, Coulomb interaction is likely to be an important scattering mechanism for SiC MOSFET device operation, resulting in very low surface mobilities. We have developed a first principles physics-based Coulomb scattering mobility model to understand this phenomenon and to study its effect on mobility in SiC devices. This type of Coulombic scattering is a quasi-two-dimensional phenomenon. Mobile charges located closer to the interface are scattered at a higher rate than those located far away from the interface. Screening of the traps and fixed oxide charges by the inversion layer mobile charges causes a decrease in the scattering rate. Also, at higher temperatures, due to a reduction in occupied trap density, and increasing energy of mobile charges, Coulombic scattering is greatly reduced. Our mobility model incorporates and accounts for all these effects. We have implemented this physics-based Coulomb scattering mobility model into a device simulator and have obtained agreement with experimental current-voltage characteristics. © 2006 American Institute of Physics. [DOI: 10.1063/1.2335673]

I. INTRODUCTION

The use of SiC has the potential of extending the microelectronics revolution into the high temperature, high-power regime. A native oxide can be grown on SiC to form a metal-oxide-semiconductor field-effect transistor (MOSFET). Since SiC has a very large band gap, high thermal conductivity, and high bulk mobility, there is a great potential for realizing high-power SiC MOSFETs with submicron dimensions. Probably, the biggest challenge in the development of SiC MOSFET devices is low surface mobility caused by excessive scattering of mobile charges at the SiC–SiO2 interface. Extremely large densities of occupied interface traps at the SiC-SiO2 interface give rise to large Coulombic scattering causing mobility degradation. In addition to this Coulomb scattering at the interface, scattering due to surface roughness and surface phonons also cause mobility degradation. In this paper, we describe a first principles Coulomb mobility model that we have developed to describe Coulomb scattering at the SiC–SiO2 interface.

Coulomb scattering of inversion layer mobile carriers by occupied traps and oxide charges is a quasi-two-dimensional (2D) phenomenon. The Coulomb potential due to the occupied traps and fixed charges decreases with distance away from the interface. So, mobile charges in the inversion layer that are close to the interface are scattered more than those further away from the interface; therefore, the Coulomb scattering mobility model is required to be depth dependent. With increase in temperature, the mobile charges gain more energy and hence are scattered less. Also, the number of occupied traps decreases with temperature; hence Coulomb scattering reduces with rise in temperature. With increase in MOSFET gate voltage, the inversion charge density increases causing increased screening of the scattering charges located at the interface; thus, Coulomb scattering away from the surface tends to decrease with gate voltage as well.

Mobility models for Coulomb scattering of electrons by impurities in a semiconductor have been described as a three-dimensional (3D) phenomenon in the literature. However, this model is not really applicable to SiC MOSFETs because here, all the scattering charges are located at the interface or some distance inside the oxide, while all mobile charges are located below the interface. The 2D Coulomb scattering described in Ref. 7 is comprehensive, but does not lend itself easily for inclusion into a drift diffusion SiC MOSFET simulator. 2D scattering by a Coulomb potential has been described for the case of sheet charge approximation of the inversion layer. Here, the authors have
calculated an average mobility. Coulomb scattering of electrons by surface oxide charges in silicon (Si)–SiO$_2$ devices has been described in literature. The authors have included the effect of distribution of electrons in the inversion layer and distribution of fixed charges inside the oxide by assuming various distribution functions for electrons and for the scattering charges, and have then calculated an average mobility in the inversion layer.

This paper describes the methodology to derive the depth-dependent quasi-2D Coulomb scattering rate and corresponding mobility from basic physics. We extend previous work by developing a mobility model that is suitable for any distribution of electrons in the inversion layer, and of fixed charges inside the oxide. It directly uses fixed charge densities at specific depths inside the oxide and the calculated mobile charge concentration at various depths inside the semiconductor to determine the scattering rate. Hence, it is easily incorporated in a 2D device simulator. Results show agreement between the measured and simulated current-voltage values for 4H-SiC MOSFETs obtained using the model.

II. COULOMB SCATTERING RATE DERIVATION

In our Coulomb scattering model, we consider scattering of mobile charges in a SiC MOSFET due to Coulombic interactions with fixed oxide charges, and with occupied interface traps at the SiC–SiO$_2$ interface. To obtain the Coulombic scattering rate due to these charges, we use Fermi’s golden rule. In this effort, we need to obtain the matrix element for Coulombic scattering. The matrix element describes the coupling between initial and final electronic states due to interactions with scattering charge centers. To obtain the matrix element for quasi-2D scattering, we first start with the 3D interaction. We then reduce the dimensionality of the system by taking the inverse Fourier transform of the 3D system over one dimension only. This gives the scattering rate in 2D as a function of depth into the device. Finally, we integrate the matrix element over all final states to obtain the depth-dependent, quasi-2D scattering rate due to trapped interface charge and oxide charge. Below we outline the important steps leading to the quasi-2D depth-dependent Coulomb scattering rate. Some additional details of the derivations are provided in the Appendix. Figure 1 shows the structure of the MOSFET device, the location of the trapped charges, oxide charges, and mobile charges, and the coordinate system used to represent them.

A. 3D matrix element

First, we obtain the matrix element for a 3D scattering case. The matrix element in Fermi’s golden rule can be written for the 3D Coulomb scattering case as

$$H_{3D} = \langle e^{-ik_{3D} \cdot r_i} | V(r) | e^{ik_{3D} \cdot r_f} \rangle = \int_V V(r) e^{i\beta_{3D} \cdot r} dr,$$

where $k_{3D}$ and $k'_{3D}$ are the wave vectors before and after the scattering event and $\beta_{3D} = k'_{3D} - k_{3D}$.

For the scattering potential, we use a 3D screened Coulomb potential function which is written as

$$V(r) = \frac{q^2}{4\pi\varepsilon_0 r} e^{-\bar{\varepsilon} r}.$$

Here, $q$ is the electronic charge and $\bar{\varepsilon}$ is the average permittivity

$$\bar{\varepsilon} = \frac{\varepsilon_{ox} + \varepsilon_{SiC}}{2},$$

where $\varepsilon_{ox}$ is the permittivity of SiO$_2$ and $\varepsilon_{SiC}$ is the permittivity of the semiconductor (SiC).

The screening wave vector $\beta_{sc}$ is given as the inverse of the Debye length for semiconductors,

$$\beta_{sc} = \sqrt{\frac{q^2N_{inv}}{\varepsilon_{SiC}Z_{av}k_BT}},$$

where $T$ is the temperature and $k_B$ is Boltzmann’s constant. $N_{inv}$ is the average 2D inversion charge density at any point along the channel of the MOSFET. $Z_{av}$ is the average depth of the inversion layer at any point along the channel of the MOSFET.

Solving the integral in Eq. (1), which looks like a 3D Fourier transform, gives (Appendix A)

$$H_{3D} = \frac{q^2}{\bar{\varepsilon} \beta_{3D}^2 + \beta_{sc}^2}.$$

This is the matrix element for the case of 3D Coulombic scattering.

B. 2D matrix element

Coulomb scattering of inversion layer charges by occupied traps and oxide charges is a quasi-2D scattering phenomenon. As all the scattering charges are located at the interface or inside the oxide, and the mobile carriers being scattered are located at different depths in the inversion layer, the Coulomb scattering potential seen by the mobile charges depends upon the distance between them and the scattering centers. Hence the inversion charges located at different distances from the interface are scattered at different rates. For writing the equations for a quasi-2D scattering phenomenon, we need to find the quasi-2D scattering matrix element $H_{2D}$.

Considering that the scattering takes place in the $X$-$Y$ plane, and is different for charges located at different depths in the $z$ direction, by taking the inverse Fourier transform of the 3D matrix element along the $q_z$ direction, we will get the matrix element for a quasi-2D scattering phenomenon.
Splitting the 3D scattering wave vector into a 2D component and a \( z \) component, we have
\[
\beta_{3D} = \sqrt{\beta_{2D}^2 + \beta_z^2}.
\]
Taking \( q_i \) in the direction of \( z \), we can write the quasi-2D matrix element by taking the inverse Fourier transform as
\[
H_{2D} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\beta_z z} d\beta_z.
\]
Solving this integral eliminates \( \beta_z \) and thereby converts the 3D scattering matrix element to 2D in wave vector space. This matrix element is especially interesting as it is explicitly dependent on \( z \) (depth) in real space. The 3D matrix element above had no real space \( z \) dependence (Appendix B).
\[
H_{2D} = \frac{q^2 e^{-q z^2/2}}{2\pi} \sqrt{\beta_{2D}^2 + \beta_{sc}^2}.
\]
This is the \( z \)-dependent quasi-2D scattering matrix element that will be used to calculate the Coulomb scattering rate.

C. Quasi-2D Coulomb scattering rate

Once we have the matrix element for the quasi-2D scattering case, we can obtain the scattering rate using Fermi’s golden rule,
\[
\Gamma_{k \rightarrow k'} = \frac{2\pi}{\hbar} |H_{2D}|^2 \delta(E_k - E_{k'}). \tag{9}
\]
Using the matrix element of Eq. (8), the quasi-2D transition rate for a unit scattering charge can be written as
\[
\Gamma_{k \rightarrow k'} = 2\pi q^4 e^{-q^2/2} \int \beta_{2D}^2 + \beta_{sc}^2 \delta(E_k - E_{k'}), \tag{10}
\]
where \( \hbar \) is Planck’s constant. \( E_k \) and \( E_{k'} \) denote the initial and final energies of the mobile charge being scattered.

Scattering of inversion layer mobile charges takes place due to Coulombic interactions with occupied traps at the interface and also with fixed charges distributed in the oxide. Defining the 2D charge density \( N_{2D}(z) \) at depth \( z \) inside the oxide as the combination of the fixed charge \( N_f \) and trapped charge \( N_t \) as
\[
N_{2D}(z) = \begin{cases} 
N_f + N_f(0), & z_i = 0 \\
N_f(z), & z_i < 0,
\end{cases} \tag{11}
\]
we can write the total transition rate as
\[
\Gamma_{k \rightarrow k'} = \frac{2\pi q^4}{4\pi \bar{E}} N_{2D}(z) e^{-2q^2/\beta_{2D}^2 + \beta_{sc}^2 \ln(z_i/z)} \delta(E_k - E_{k'}). \tag{12}
\]
The quasi-2D scattering rate is given as
\[
\frac{1}{\tau(z, z_i)} = \frac{1}{4\pi^2} \int_{k'=0}^{\infty} \int_{\theta=0}^{\pi} \Gamma_{k \rightarrow k'}(1 - \cos \theta) d\theta dk', \tag{13}
\]
where \( \tau(z, z_i) \) is the transport relaxation time, which represents the average time between successive scattering events of an electron located at a depth \( z \) inside the semiconductor, due to Coulombic interaction with some charge located at depth \( z_i \) inside the oxide. \( \theta \) is the scattering angle between the initial state \( k \) and the final state \( k' \).

To obtain the total scattering rate for an electron with wave-vector \( k \), at depth \( z \) inside the semiconductor, due to a Coulomb center a distance \( z_i \) into the oxide from the interface we evaluate Eq. (13). We first substitute the transition rate given by (12) into Eq. (13). We then express the 2D scattering vector \( \beta_{2D} \) in terms of \( k' \) and the scattering angle \( \theta \) This allows us to then change our integration variable \( k' \) to energy (\( E \)). As the integral involves a delta function in energy, and as Coulomb scattering is elastic, the integration in energy space reduces to an expression involving the energy of the electron. Thus we get an energy dependent scattering rate. This derivation requires a moderate amount of mathematics, the details of which we include in the Appendix. Here we write the final result which is the energy dependent scattering rate as (see Appendix C for details)
\[
\frac{1}{\tau(z, z_i, E)} = \frac{q^2 N_{2D}(z)}{16 \pi \bar{E} \hbar} F(z, z_i, \beta_{sc}), \tag{14}
\]
where
\[
F(z, z_i, \beta_{sc}) = \int_{0}^{\pi/2} \int_{0}^{2 \pi} \left[ 1 - \frac{\beta_{sc}^2}{(8m^*E/\hbar^2)\sin^2 \alpha + \beta_{sc}^2} \right] \times \exp \left[ -2 \sqrt{\frac{8m^*E}{\hbar^2} \sin^2 \alpha + \beta_{sc}^2} (z - z_i) \right] d\alpha. \tag{15}
\]
Equation (15) gives the rate at which a mobile carrier having an energy \( E \) and located at a depth \( z \) inside the semiconductor will be scattered by scattering charges of density \( N_{2D} \) located at a distance \( z_i \) inside the oxide. For a SiC MOSFET, \( z_i \) starts from the interface (\( z_i = 0 \)) and extends 30–50 Å into the oxide. \( F \) is the form factor that describes the dependence of the Coulombic scattering rate on the amount of screening (\( \beta_{sc} \)), and on the distance between the mobile charge and the scattering charge (\( z - z_i \)). The amount of screening depends upon the density and the average thickness of the inversion layer. \( F \) can reach a maximum of \( \pi/2 \), and approaches zero for electrons that are very far from the interface. \( F \) is further described later in the paper.

III. QUASI-2D COULOMB MOBILITY

In order to implement this scattering model in a device simulator, we need to convert the scattering rate equation to a mobility equation. In this section we show how we obtain the depth-dependent mobility from the scattering rate of Eq. (15). We then discuss some physical attributes of the resulting mobility model.

A. Mobility derivation

The average scattering rate \( \langle 1/\tau \rangle \) is calculated by approximating the average energy of a mobile charge as \( E = k_B T \). Using this we can write the average scattering rate as
\[
\left\langle \frac{1}{\tau} \right\rangle = \frac{q^2 N_{2D}(z_i)}{16 \pi \varepsilon_0^2 h k_B T} F(z_i, \beta_{sc}),
\]
where the form factor \( F \) becomes
\[
F(z_i, z_{\text{sc}}, \beta_{sc}) = \int_{\alpha=0}^{\pi/2} \left[ 1 - \frac{\beta_{sc}^2}{(8m^* k_B T h^2 \sin^2 \alpha + \beta_{sc}^2)} \right] \times \exp \left[ -2 \sqrt{\frac{8m^* k_B T h^2}{h^2} \sin^2 \alpha + \beta_{sc}^2} (z - z_i) \right] d\alpha.
\] (17)

We write the mobility from the average scattering rate as
\[
\frac{1}{\mu_c(z_i, \beta_{sc}, T)} = \frac{m^*}{q} \left\langle \frac{1}{\tau} \right\rangle.
\] (18)

Thus, finally, we have the equation for the mobility of a carrier present at depth \( z \) inside the semiconductor due to Coulombic scattering from charges at \( z_i \) inside the oxide as
\[
\frac{1}{\mu_c(z_i, z_{\text{sc}}, T)} = \frac{m^* q^2 N_{2D}(z_i)}{16 \pi \varepsilon_0^2 h k_B T} F.
\] (19)

The total Coulomb mobility for a mobile carrier at a depth \( z \) can be obtained by summing up the scattering rates due to scattering charges located at all \( z_i \)'s, using Matthiessen’s rule as
\[
\frac{1}{\mu_c(z, T)} = \sum_{z_i} \frac{1}{\mu_c(z_i, z_{\text{sc}}, T)}.
\] (20)

**B. Discussion of mobility model**

The Coulomb scattering mobility model developed above incorporates some interesting physics of the inversion layer. The notable points of this model are summarized below.

1. The Coulomb scattering rate is directly proportional to the density of occupied interface traps and fixed oxide charges.
2. Fixed oxide charges located away from the interface deeper into the oxide have less effect on the scattering of inversion layer electrons.
3. Mobile charges located away from the interface and inside the semiconductor are scattered less as compared to those located near the interface. Thus, the distribution of mobile charges inside the inversion layer plays an important role in determining the total mobility and, thereby, the total current in the device. The model works for and can be applied to any arbitrary distribution of charges inside the device.
4. The effect of screening of the scattering charges (occupied traps and fixed oxide charges) by inversion layer mobile charges has been included in the mobility model as part of the form factor \( F \). The model shows that with increase in inversion layer charge, the screening increases (\( \beta_{sc} \) increases), causing the total Coulomb mobility to increase.
5. Temperature dependence of Coulomb mobility is obtained implicitly from the model. For SiC MOSFETs, the mobility increases with increase in temperature due to two effects. First, the mobility is directly proportional to temperature which indicates that mobile charges with more energy are scattered less, thereby increasing mobility with rise in temperature. Secondly, the interface trap density \( N_d \), which is part of the 2D scattering charge density \( N_{2D} \), decreases with increase in temperature, causing an increase in Coulomb mobility.

6. In the specific case when it is assumed that the interface trapped charge and the fixed oxide charge is located only at the 4H-SiC/SiO₂ interface \( (z_i=0) \), the depth-dependent Coulomb mobility may be given as
\[
F = \int_{\alpha=0}^{\pi/2} \left[ 1 - \frac{\beta_{sc}^2}{(8m^* k_B T h^2 \sin^2 \alpha + \beta_{sc}^2)} \right] \times \exp \left[ -2 \sqrt{\frac{8m^* k_B T h^2}{h^2} \sin^2 \alpha + \beta_{sc}^2} (z - z_i) \right] d\alpha
\] (21)

and
\[
\frac{1}{\mu_c(T)} = \frac{m^* q^2 (N_f + N_d)}{16 \pi \varepsilon_0^2 h k_B T} F.
\] (22)

7. In the classic sheet approximation for the inversion layer, it is assumed that the inversion layer is an infinitesimally thin sheet of charge located at the interface. For this approximation, the form factor can be written as
\[
F^{\text{sheet}} = \int_{\alpha=0}^{\pi/2} \left[ 1 - \frac{\beta_{sc}^2}{(8m^* k_B T h^2 \sin^2 \alpha + \beta_{sc}^2)} \right] d\alpha.
\] (23)

8. For the classical sheet approximation and with no screening,
\[
F^{2D} = \frac{\pi}{2}
\] (24)

and
\[
\frac{1}{\mu_c(T)} = \frac{m^* q^2 (N_f + N_d)}{32 \varepsilon_0^2 h k_B T}.
\] (25)

**IV. IMPLEMENTATION**

We implemented this Coulomb scattering mobility model in our drift-diffusion device simulator that was originally written for 6H-SiC MOSFETs and was described previously in this journal. The simulator includes detailed mobility models for bulk phonon scattering, surface phonon scattering, and surface roughness scattering. Here we adapt the simulator for 4H-SiC MOSFETs and incorporate the Coulomb scattering mobility model described above. Our simulations for the drain current \( I_D \) versus gate voltage \( V_{GS} \) curves match the experimentally measured data very well. Figures 2 and 3 show the comparison of simulated and experimentally measured room temperature \( I_D-V_{GS} \) curves for a 10 \( \mu \)m long and 440 \( \mu \)m wide \( n \)-channel 4H-SiC MOSFET. The simulations match the experiment in the subthreshold, near threshold, and linear regions.

Mobility variation along the channel and with depth away from the interface at a gate voltage of 2 V is shown in
V. CONCLUSION

A physics-based Coulomb scattering mobility model for scattering of inversion layer mobile carriers by occupied interface traps and fixed oxide charges in SiC MOSFETs has been presented. The mobility model takes into account scattering by occupied traps and by fixed oxide charges, screening of these scattering charges by inversion layer mobile charges, any arbitrary distribution of mobile charges in the inversion layer, depth dependence of Coulomb mobility, and temperature effect on scattering. The model is suitable for implementation in a device simulator.

ACKNOWLEDGMENTS

The authors are grateful to F. B. McLean, Charles J. Scozzie, and Dan Habersat for significant contributions to the investigation.

APPENDIX A: 3D SCATTERING MATRIX ELEMENT

Here we describe how we derive the 3D scattering matrix element by solving the 3D Fourier transform [Eq. (1)].

\[
H_{3D} = \langle e^{-ik_3 r}\rangle V(r) e^{ik_3 r} = \int_{r} V(r)e^{i\beta_{3D} r}dr. \tag{A1}
\]

Substituting for \( V(r) \) from Eq. (2), we can write the integration using spherical coordinates as

\[
H_{3D} = \frac{q^2}{4\pi\varepsilon} \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \left( \frac{1}{r} e^{-\beta_{3D} r} e^{i\beta_{3D} r} \cos \theta \right) \\
\times r^2 \sin \theta d\theta d\phi dr. \tag{A2}
\]

Using change of variable \( t=\cos \theta \),

\[
H_{3D} = \frac{q^2}{2\varepsilon} \int_{r=0}^{\infty} r e^{-\beta_{3D} r} dr \int_{t=-1}^{1} e^{i\beta_{3D} t} dt, \tag{A3}
\]

\[
H_{3D} = \frac{q^2}{2\varepsilon} \beta_{3D}^{-1} \int_{r=0}^{\infty} r e^{-\beta_{3D} r} dr \left( e^{i\beta_{3D} t} - e^{-i\beta_{3D} t} \right), \tag{A4}
\]

\[
H_{3D} = \frac{q^2}{\varepsilon \beta_{3D}} \int_{r=0}^{\infty} e^{-\beta_{3D} r} \sin(\beta_{3D} r) dr, \tag{A5}
\]

\[
H_{3D} = \frac{q^2}{\varepsilon \beta_{3D}} \int_{r=0}^{\infty} \left( e^{-\beta_{3D} r} \frac{-\beta_{3D} \sin(\beta_{3D} r)}{\beta_{3D}^2 + \beta_{3D}^2} \right) dr. \tag{A6}
\]

On applying the limits we get

\[
H_{3D} = \frac{q^2}{\varepsilon \beta_{3D}} \frac{1}{\beta_{3D}^3 + \beta_{3D}^2}. \tag{A7}
\]

This is the 3D scattering matrix element as given in Eq. (5).
APPENDIX B: QUASI-2D SCATTERING MATRIX ELEMENT

Here we show the steps to solve the inverse Fourier transform of Eq. (7) and obtain the quasi-2D scattering matrix element of Eq. (8).

From Eq. (7), we have
\[ H_{2D} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_{3D} e^{i\beta z} d\beta. \]  

(B1)

Substituting for \( H_{3D} \) from Eq. (8) and writing \( \beta_{2D} = \beta_{2D}^1 + \beta_{2D}^2 \), we have
\[ H_{2D} = \frac{q^2}{2\pi e} \left[ \frac{1}{\beta_{2D}^1 + \beta_{2D}^2} \right] e^{i\beta z} d\beta. \]

We solve this complex integral using the residue method by considering the complex contours \( c_1 \) and \( c_2 \) shown in Fig. 5.

The complex integral can be solved as
\[ H_{2D} = \frac{q^2}{2\pi e} \text{Res} \left( \frac{e^{i\beta z}}{\beta_{2D}^1 + \beta_{2D}^2} \right)_{\beta = i(\beta_{2D}^1 + \beta_{2D}^2)}. \]

(B4)

\[ H_{2D} = \frac{q^2}{2\pi e} \text{Res} (e^{-\sqrt{\beta_{2D}^1 + \beta_{2D}^2} x} / 2i(\beta_{2D}^1 + \beta_{2D}^2)). \]

(B5)

\[ H_{2D} = \frac{q^2}{2\pi e} e^{-\sqrt{\beta_{2D}^1 + \beta_{2D}^2} x} / \sqrt{\beta_{2D}^1 + \beta_{2D}^2}. \]

(B6)

This is the depth-dependent quasi-2D scattering matrix element of Eq. (8).

APPENDIX C: QUASI-2D COULOMB SCATTERING RATE

Here we show the steps followed to obtain the quasi-2D depth-dependent Coulomb scattering rate of Eq. (14) from the momentum relaxation rate of Eq. (13).

From Eq. (13), we have the momentum relaxation rate as
\[ \frac{1}{\tau(z, z') = 1 - \frac{1}{4\pi^2} \int_{k'=0}^{\infty} \int_{\theta=0}^{\pi} \Gamma_{k'=k'}(1 - \cos \theta) d\theta dk'. } \]

(C1)

Substituting for \( \Gamma_{k'=k'} \) from (12), we have
\[ \frac{1}{\tau(z, z') = \frac{q^4 N_{2D}(z)}{2\pi^2 e^2} \times \int_{k'=0}^{\infty} \int_{\alpha=0}^{\pi/2} \frac{e^{-2v(k')^2 \sin^2 \alpha + \beta_{2D}^2}(z - z')}{4k'^2 \sin^2 \alpha + \beta_{2D}^2} \times k' \delta(E_k - E_{k'}) \sin^2 \alpha d\alpha dk'. } \]

(C2)

Here, \( \beta_{2D} \) represents the 2D scattering wave vector and \( \theta \) is the scattering angle, as shown in Fig. 6.

As Coulomb scattering is elastic, \( |k'_{2D}| = |k_{2D}| = k \) and hence
\[ \beta_{2D} = k'_{2D} - k_{2D} = 2k' \sin \frac{\theta}{2}. \]

(C3)

Also writing \( \alpha = \theta/2 \),
\[ \frac{1}{\tau(z, z') = \frac{q^4 N_{2D}(z)}{2\pi^2 e^2} \times \int_{k'=0}^{\infty} \int_{\alpha=0}^{\pi/2} \frac{e^{-2v(k')^2 \sin^2 \alpha + \beta_{2D}^2}(z - z')}{4k'^2 \sin^2 \alpha + \beta_{2D}^2} \times k' \delta(E_k - E_{k'}) \sin^2 \alpha d\alpha dk'. } \]

(C4)

Writing the energy at \( k' \), we apply a change of variable from \( k' \) to \( E_{k'} \),
\[ E_{k'} = \frac{\hbar^2 k'^2}{2m^*} \Rightarrow k' dk' = \frac{m^*}{\hbar^2} dE_{k'}. \]

(C5)

where \( m^* \) is the effective mass of the mobile charge. Substituting for \( k' \) and noting that the integral involves a delta function in \( E_{k'} \), the scattering rate equation simplifies to
\[ \frac{1}{\tau(z, z', E) = \frac{q^4 m^* N_{2D}}{2\pi^2 \hbar^3} \times \int_{\alpha=0}^{\pi/2} \frac{e^{-2v[(8m^*E/h^2)\sin^2 \alpha + \beta_{2D}^2](z - z')}}{(8m^*E/h^2)^2 \sin^2 \alpha + \beta_{2D}^2} \times \sin^2 \alpha d\alpha. } \]

(C6)

We write the energy dependent, and depth-dependent scattering rate as
\[
\frac{1}{\tau(z, z_i, E)} = \frac{q^4 N_{2D}(z_i)}{16\pi \varepsilon^2 \hbar} F(z, z_i, \beta_{sc}),
\]
(C7)

where
\[
F(z, z_i, \beta_{sc}) = \int_{0}^{\pi/2} \left[ 1 - \frac{\beta_{sc}^2}{(8m^*E/\hbar^2)\sin^2 \alpha + \beta_{sc}^2} \right] \exp \left[ -2 \sqrt{\frac{8m^*E}{\hbar^2} \sin^2 \alpha + \beta_{sc}^2} (z - z_i) \right] d\alpha
\]
(C8)
gives the dependence of Coulomb scattering on the amount of screening (\(\beta_{sc}\)) and the distance between the mobile charge and the scattering charge center (\(z - z_i\)).